The Relationship of the Concept of Fair to the Construction of Probabilistic Understanding

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This report combines the results of two small studies of probability based on the concept of fair. The outcomes of the first study led to additional opportunities in the second and finally to a hypothesis of conceptual development. In both studies students from grades 3 to 9 were interviewed using a protocol designed to assess their understanding of fair in relation to dice. The theoretical framework used to analyse student responses was the SOLO Model with Multimodal Functioning developed by Biggs and Collis.

Children experience the concept of fairness in a variety of contexts most of which are outside the school curriculum. Two consecutive studies discussed in this paper aimed to associate these experiences with the development of a mathematical notion of fairness as it relates to probability. In particular the investigations focussed on the notion of fairness as it relates to dice and equally likely outcomes. An understanding of equally likely outcomes is an important aspect in the development of an understanding of probability (Australian Education Council, 1994). Determining mathematical fairness can be a higher order cognitive task because not only does it require knowledge of equally likely events but also in the context of the activities used in the second study it requires an understanding of sampling and data analysis.

Many stochastic understandings begin development at an early age (Yost, Siegel & Andrews, 1962; Goldberg, 1966; Fischbein, 1975) and with the concept of fair it would be reasonable to assume that children begin to develop their understanding before they start school. Sibling and family rivalry provides a motivation for understanding the meaning of fair, with children having many opportunities to construct their meanings in relation to sharing and equality (Streefland, 1991). It is doubtful that to date the full extent of these understandings have been explored. As a consequence children may be making meaning of school-presented material in the context of their current understanding of fair and this may lead to a mismatch between the intended learning planned by the teacher and the actual learning constructed by the students (Kapadia, 1988). Although not specifically related to fairness the research of Fischbein and Gazit (1984) describing the intuitions developed by relatively young children through experiences prior to formal schooling indicates a connection between these intuitions and the later development of complex mathematical concepts. They stress the importance of acknowledging these intuitions or cognitive beliefs because they can assist or impair understanding of probability and statistics concepts during schooling.

Very little appears to have been written about the development of the mathematical concept of fairness although an expectation that students have an understanding of the concept of fairness is implicit in a number of studies both of older students (Konold, 1989; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993) and of younger students (Watson, Campbell & Collis, 1993). Konold, et al.'s research (1993) poses questions about the likelihood of possible outcomes when a fair coin is tossed five times. Fairness associated with games has been the focus of numerous projects (Bright, Harvey & Wheeler, 1981; Peard, 1995).

Kahneman and Tversky (1972) described a representativeness heuristic which students employed when asked to predict randomly generated outcomes. For example, if the students were asked to predict the outcomes from tossing a regular die sixty times, using the representativeness heuristic their responses would closely resemble the theoretical outcome of 10 of each number 1 to 6. In exploring the notion of fairness the second study reported in this paper has focussed on the students' prediction of outcomes from a similar perspective and then extended this idea to explore how far results differ from the uniform theoretical distribution before students are willing to accept that the sample is not representative.

A consistent theme in research exploring students' understanding of probability and statistics has been that of students' misconceptions (Kahneman & Tversky, 1972; Shaughnessy, 1977; Cox & Mouw, 1992). The use of this term indicates a deficit view of learning which judges mental constructs as being either correct or incorrect. The authors prefer a constructivist perspective of learning where the student's understanding is not viewed as a misconception but rather as an alternative conceptualisation to that which has been conventionally agreed. Some "misconceptions" could more accurately be described as understandings which are incomplete in some respect.

Theoretical Framework

The theoretical model used for the analysis of the data from these studies is the SOLO Taxonomy (Biggs & Collis, 1982) incorporating intermodal and multimodal functioning (Biggs & Collis, 1991; Collis & Biggs, 1991). Evolving from Piagetian theory, the model postulates five modes of functioning which originate in a fixed order but which continue to develop alongside each other throughout life: sensorimotor (from birth), ikonic (from early childhood), concrete symbolic (from the years of schooling), formal (for those capable of higher education), and post formal (associated with research). The ikonic and concrete modes will be most relevant in these studies because they are associated respectively with intuitive functioning and with the concrete symbolic learning which takes place in school often based on concrete materials.

The model proposes that earlier developing modes continue to develop in conjunction with later modes and provide opportunities for interaction which may facilitate intellectual functioning in general. Results demonstrating multimodal functioning in the acquisition of understanding of fractions (Watson, et al., 1993) and the work of others in the field of stochastics (Berenson, Friel & Bright, 1993; Callingham, 1993; Fischbein & Gazit, 1984; Watson & Collis, 1993) lead to the belief that such functioning will also occur for concepts related to fairness in probability.

Within each mode hierarchical development takes place by means of a cycle of learning (Biggs & Collis, 1982; 1991) which has five levels: prestructural, unistructural, multistructural, relational and extended abstract. Considering responses within a mode of functioning, it is the middle three types of response which are of main concern: unistructural (U), multistructural (M), and relational (R). Recent studies of mathematical understanding have found two U-M-R cycles operating within the concrete symbolic mode in connection with students' understanding of volume measurement (Campbell, Watson & Collis, 1992), fractions (Watson, et al., 1993), arithmetic mean (Callingham, 1993) and data handling (Watson, Collis, Callingham & Moritz, in press). In complex settings, mathematical fairness is a higher order concept which incorporates aspects of both probability and statistics and therefore is likely to require two U-M-R cycles to categorise responses in the concrete symbolic mode. With the concept of fairness intuitive understandings would be expected to be significant because children probably develop notions of fairness from an early age through varied experiences with sharing and games. It has been noted that intuitive reasoning is frequently used by adults in solving problems with probabilistic contexts (Konold, et al., 1993). There is hence an interest in the ikonic mode and the interaction of it with the concrete symbolic mode.

Study 1

Method

An interview protocol was used in the first study with 30 girls in grades 3, 5, 7 and 9 from a South Australian independent school. The dice protocol was one of eight used during a 45-minute interview. Students were introduced to the protocol with questions on whether they played games with dice and whether some numbers were more likely to come up more often than others. Then they were shown three dice; a blue one which had been weighted producing a bias in favour of the number two, an unmodified red one which was similar in external appearance to the blue die, and a white one on which two of each of the numbers 1, 2 and 3 were represented. In an ensuing discussion, students were asked whether the dice were fair or not and how they could justify their decisions.

Results

An analysis of the results from the South Australian sample produced several trends which were significant in the design of the second study and the length of time devoted to the topic of fairness of dice. The results are first summarised with respect to the type of argument used in discussing outcomes expected in relation to tossing dice.

In grade 3 all responses incorporated ikonic belief at some stage. These were related to personal experience (e.g., "whenever I get a 1, I knew I was getting a 1 which is sometimes a pain), myths (e.g., "3 because in fairy-tales there is 3 wishes and there is 3 fairy god-mothers and there's 3 witches and there's all sorts of 3 things"), anthropomorphism (e.g., "the dice isn't really a genius") and a combination of these (e.g., "6 comes up not the most because it's biggest and for me 2 usually comes up"). These responses decreased markedly after grade 3 with only one grade 5 emphasising similar beliefs.

Arguments based on physical characteristics, unsystematic trials (a couple of tosses) or an "anything can happen" view of chance were relatively constant across all grades in the sample. These were hypothesised to be typical of the first cycle concrete symbolic mode (Watson, Collis & Moritz, 1995a). They were often combined with ikonic support and more sophisticated arguments. With reference to physical characteristics, reference to colour decreased and heaviness increased as criteria for determining fairness over the grades interviewed.

By grade 7 half of the students were suggesting systematic trials to determine fairness of dice and by grade 9 most students were adding mathematical arguments to justify their reasons for drawing conclusions about dice. This indicates movement to the second cycle of the concrete symbolic mode. The suggestion of trials in various forms, however, was not restricted to the oldest students. One grade 3 suggested that to determine which dice were fair, "You could play a game all day and see what happens." A grade 5 student said, "You need heaps of throws." Older students made more sophisticated suggestions.

Of particular interest were the responses which dealt directly with the idea of dice being "fair", that is each number having the same chance of coming up. The youngest students were willing to accept both the view that some numbers come up more than others and the view that each number has the same chance of coming up. This may be a result of viewing "chances" as associated with the future and hence "fair", while numbers which seemed to come up more often were observations from personal experience in the past and hence factual evidence. Early childhood teachers will recognise that this phenomenon of being able to hold contradictory propositions as true at the same time is fairly typical of young students' views on the mathematical underpinning of everyday experiences.

Older students were split between two categories of understanding in relation to these questions. In one group were those who believed that no numbers were more likely to come up than others and hence all numbers have the same chance on a fair dice. Whether this understanding was based on personal experience of playing games, experience of tossing dice, or a theoretical understanding of the mechanism associated with the construction of dice is unknown. In the other group were students who acknowledged that, in one way or another, it may appear that some numbers come up more often when dice are tossed. This may be due to some personal experience, say wanting a number and it not occurring. These students, however, believed that in the long run, say if many trials were performed, the results for all numbers would even out and hence fair dice produce the same chance for all numbers. These students were dealing with conflict of sometimes-observed short term results and the understanding of long term trends. It is hypothesised that this represents a more sophisticated understanding of the operation of fair dice.

For those who did not suggest the using of a series of trials to determine which dice were unfair, physical characteristics were the main alternative. Once it was realised that one of the dice had the numbers 1, 2, 3 repeated, it was not uncommon for younger students to declare the other two dice "fair" because they had all of the numbers from 1 to 6 on them. Some (not many) suggested that the die with two each of 1, 2, 3 was still fair because it had two of each number - this was certainly a real possibility and represents a sophisticated intermediate response - the use of trials being an independent suggestion.

Within the scope offered in Study 1, the responses were classified as shown in the left column of Table 1. In the ikonic mode, no distinctions were made in terms of a U-M-R cycle. In the first cycle of the concrete symbolic mode, unistructural responses reflected single physical characteristics or the "anything can happen" view. Also contradictions were accepted. At the multistructural level, other characteristics of the dice were combined to judge either fairness or unfairness (e.g., the distribution of numbers). At the relational level, some initial rather unsophisticated attempts to consider pre-sampling ideas occurred as did the realisation of fairness in other situations than the presence of the numbers from 1 to 6. More sophisticated ideas related to sampling occurred in the second cycle but there was a ceiling effect on the opportunity for responses due to the protocol's structure and the length of time available.

Study 2

Method

During the analysis of Study 1, there were indications that given the opportunity students would have provided more elaborate responses. This led to modification in the design of the protocol used in Study 2 and enabled a more detailed categorisation of responses in the ikonic mode.

In Study 2, 24 Tasmanian students from grades 3, 6 and 9 participated in a similar interview protocol incorporating the biased blue die and the red die. The expanded dice protocol occupied the entire 45-minute interview. A "Horse Race" game was used to provide a context for questioning the students about whether the dice were fair, and a series of graphs were also used for the same purpose. The data presented in the graphs were produced using Prob Sim© (Konold & Miller, 1992) where three of the dice were fair and three of the dice were unfair. The data were selected to show a range of samples of 60 trials where one was fair and looked fair, two were fair but looked unfair, one was unfair but looked fair, and two were unfair and looked unfair. See Lidster, Pereira-Mendoza, Watson and Collis (1995) for a complete set of the series of graphs.

Results

The opportunity to explore ideas more fully in Study 2 made possible a more detailed analysis of responses in terms of the SOLO model. The responses that were categorised as ikonic were identified within three levels: unistructural, "all the dice in the whole wide world are fair!"; multistructural, "the die is fair for me but it wouldn't be fair for the person I was playing against"; and relational, "in some games six is the worst number to roll." The unistructural responses asserted a single belief, whereas the multistructural responses contained two or more connected, but not integrated, beliefs and often considered perspectives other than the student's own. When past experience influenced judgements the responses were relational within the ikonic mode.

A shift from beliefs and views held from experience in the past to an attention to the present situation marks the transition from ikonic responses to unistructural responses in the first cycle of the concrete symbolic mode. When asked how they would determine if a die were fair, many of the grade 3 to 6 students attended to the physical characteristics of the die: "There aren't any chunks out of it," or "The bits missing where the dots are might make it unfair." Other responses classified in this category related to the way that the die is rolled and these came from students across all grades. Students who connected several of the unistructural ideas which were related to physical characteristics of the dice were considered to have responded at a multistructural level.

Many of the alternative or incomplete conceptualisations of fair appeared within the first cycle of the concrete symbolic mode. Some students displaying this level of cognition were selecting particular outcomes *between* two samples. Figure 1 depicts the outcomes from the blue and red die after the completion of a "Horse Race" game. When students were comparing the samples produced by the two dice after playing the Horse Race game, a grade 3 student, ignoring the fact that some numbers came up more often than others, established that "the dice are fair because they have about the same amount [respectively] of each number." Another student said, "The fives look about the same for both dice and so do the ones." After comparing the number of times one came up on the blue and red die a grade 6 student maintained, "One is the fairest of them all".

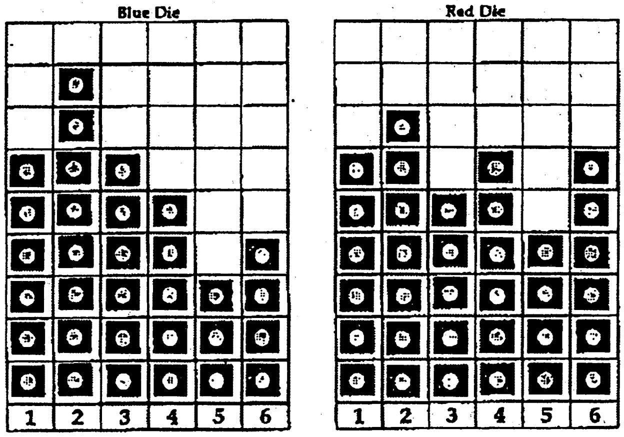

Figure 1. Results of a "Horse Race" game.

Other students compared the outcomes *within* a sample from a single die when looking at the graphs. "This dice is fair because the 1 and 2 are fair and the 3, 4, 5 & 6 are all fair." Some students encountered conflict at this level deciding that a die could be both fair and unfair at the same time. "Fair in one way but not in another way. Basically really fair but it isn't fair for the four," or "This one looks pretty fair except for the two." These responses are multistructural with ikonic support in the form of a belief that dice must be fair. This belief together with a valuing of some numbers over others also led to responses which judged a die to be partially fair. In comparing graphs exhibited by the interviewer, one student commented, "Pink [the pink die] is a bit unfair because 1 and 2 are up but 3, 4, 5 & 6 are a bit fair."

An emerging understanding of sampling appears at the relational level of the first cycle of the concrete symbolic mode. Responses at this level demonstrate a slight shift from the multistructural level with an acknowledgment that a sample from a fair die may look unfair. "Looks pretty unfair but they could be fair." Several grade 6 students in grappling with the notion of trials suggested that the game could be played over and over again to see if the dice were fair. These were also considered relational responses in that they were constructing an integrated scheme by which fairness could be determined.

It is the lack of a more explicit explanation of trialing which differentiates the first cycle relational level responses from the second cycle unistructural level responses in the concrete symbolic mode. At the higher level students described the process of trials easily, demonstrating a consolidated idea of using a sampling process to determine fairness. At the second cycle multistructural level the students considered two or more samples in their judgement of the die but failed to suggest or recognise that a larger sample would provide more reliable information. At the relational level the responses resolved conflict by considering trends across a number of samples of increasing size.

The responses from Study 2 are summarised by SOLO level in the right column of Table 1. Generally at an ikonic level the students relied upon personal experience and belief to judge whether or not a die was fair. The U-M-R cycle within the ikonic mode which was identified in the second study classified responses which began with personal feelings and a view of the die as it related to them personally (U). This was followed by a shift to a consideration of how others might view the die but still within a personal view of the die as it behaved relative to the one using the die (M). Finally the responses referred to past experiences playing games or using dice (R). These responses incorporated experiences of being expected to accept what appeared to be unfair because that was life and "sometimes you had to lose". With respect to Study 1, it is likely that anthropomorphic and mythical responses are at the unistructural level, unless combined together in a multistructural way.

Of particular interest in the first cycle of the concrete symbolic mode was a collection of alternative conceptual frameworks identified at the multistructural level.

These became evident when students were struggling to decide whether results from tossing dice were sufficiently the same or different. It was obvious that for most students this was the first time that they had been forced to consider the possibilities of dice being unfair and what this might look like. This was significant in contributing to cognitive conflict and gave rise to three identifiable alternative conceptualisations. The first was the notion of a die which could be both fair and unfair at the same time. For example, it could be fair for some of the numbers on the die but it was unfair for another or others. The second alternative conceptualisation also related to the die being both fair and unfair but this referred to separate occasions. The die could be judged fair for one sample of tosses but on another occasion the same die would be unfair. The third alternative conceptualisation was of a fairness continuum. A die could be fairer than another but not as fair when compared to a third die. This continuous property of fairness was also applied when comparing samples from the same die, i.e., "this die is getting fairer" or "it is not as fair this time." This illustrates typical multistructural responses as they develop towards the relational level, where the conflict is resolved, for the time being at least, by some form of higher level integration.

In the second U-M-R cycle, the students began to use the idea of sample more effectively as a way of judging a die to be fair or unfair. The observed constructions of this understanding became more structurally complex at each level and at the relational level an understanding based on concrete sampling experience was complete. This understanding acknowledged increasing confidence with increasing sample size and was particularly evident in older students' interpretations of the computer-generated graphs.

Table 1.
SOLO levels for the concept of fair identified in each study

<table>
<tr><th colspan="2">SOLO Level</th><th>Study 1</th><th>Study 2</th></tr>
<tr><td rowspan="3">Ikonic</td><td>U_1</td><td rowspan="3">Anthropomorphic dice
Witches and myths
Personal experiences</td><td>Me
• what I want / don't want
• situation specific</td></tr>
<tr><td>M_1</td><td>Others
• what people want</td></tr>
<tr><td>R_1</td><td>Experiences
• with games
• that's life
• some numbers are harder to roll</td></tr>
<tr><td rowspan="3">Concrete Symbolic (First Cycle)</td><td>U_1</td><td>Edges
Anything can happen
Fair and unfair both.</td><td>How this happens
• physical properties
• the way it is rolled</td></tr>
<tr><td>M_1</td><td>Looks normal: all six numbers
Unfair - only 1, 2, 3.
"All" dice fair: same chance</td><td>Comparison between and within a dice
• Fair/Fairer/Fairest
• Both Fair & Unfair
(Response with/without ikonic support)</td></tr>
<tr><td>R_1</td><td>Two of each 1, 2, 3 - fair.
"Play games all day."
Dice may appear unfair but be fair in the long term</td><td>Looks unfair but could be fair.
(Response with/without ikonic support)
Could play game over and over</td></tr>
<tr><td rowspan="3">Concrete Symbolic (Second Cycle)</td><td>U_2</td><td>Suggest trials</td><td>Could do 1000 tosses, use tally marks</td></tr>
<tr><td>M_2</td><td>Carry out limited systematic trials</td><td>Information from two or more samples put together but lack of recognition that generally a larger sample is more reliable predictor.</td></tr>
<tr><td>R_2</td><td></td><td>Resolves conflict when presented with results of trials within concrete context of the graphs</td></tr>
</table>

Discussion

There are some interesting trends in the data from both studies which are consistent with the development of other probability and statistics concepts such as likelihood and sampling (Watson, Collis & Moritz, 1995b). It is hypothesised that there are two distinct themes in the conceptualisation of fair. An emerging understanding of what it means for a die to be fair is followed by an appreciation of the significance of sampling in determining if a die is fair. At the relational level of the first cycle of the concrete symbolic mode these conceptualisations overlapped when some students displayed more complex mathematical thinking when they suggested playing the game over and over to determine if the die were fair. They appeared to have not yet acquired vocabulary to adequately describe their intentions in terms of sampling and trials. The vocabulary was used with ease, however, in student responses at the unistructural level of the second cycle and above.

It was observed that most students in the first study had very little idea of how to test whether the dice were fair and in the second study most were willing to accept that a die was fair in the face of strong evidence that it was most probably biased. A second hypothesis was based on the students' explanations when they appeared to have no experience or awareness of how a die could be made to be unfair. These students could not reconcile two conflicting situations: (1) a *belief* that it is not possible to make a die behave unfairly and (2) an *observation* that this sample looks odd because, for example, you would not expect the "4" to come up that often. Added to this were students' experiences of playing games and rolling dice which produce unequal outcomes even though the dice are *known* to be fair. Hence the conclusion that many students reached was that the results of tossing the die looked strange but the die was probably fair.

Origins of the notion of fair are probably established in experiences with sharing and games played at home where parental explanations for losing would be designed to convince the child that the dice can come up with any results and that it is still fair and acceptable. This, together with a lack of knowledge about the possibility that a die could be unfair or how that could be achieved, provides strong support for the development of alternative and incomplete conceptualisations of fairness. The implications for teaching which arise from this observation are discussed elsewhere (Lidster, et al., 1995).

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